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# Clarifying assumptions behind the empirical testing of a physiologically plausible computational model of rhythm perception

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## Introduction

The paper "Empirical testing of a physiologically plausible computational model of rhythm perception" (Angelis, 2010) proposed an examination of the accuracy of Edward Large's computational model to simulate aspects of human's rhythm perception. In that paper, a series of experiments were proposed to address the above proposition. A detailed description of these experiments can be found in the methodology section of the above document. The present paper seeks to clarify various cross-disciplinary issues from the different disciplines involved, such as musicology, neurophysiology and mathematics. The issues to be clarified are the following:

a) The musical reality or realities that the computational model addresses or attempts to account for.

b) The physiological realities the model presumes (for instance details concerning the network of neural oscillators, the relationships between oscillators, and what happens when the brain confronts the musical situations documented in (a) above.

c) An explanation of the distinctions between linear systems and nonlinear systems and the relevance of this to how we understand firstly, the 'brain on music' and secondly, the computational model.

d) The ways in which these inform the mathematical model (the mathematics of the model need not to be explained fully, but the musical and physiological elements folded into that model do).

## 1.Musical Realities: Rhythm, Beat and Meter

This section describes the musical realities that the computational model assumes in order to simulate aspects of rhythm perception. These are the rhythm, the beat and the meter. The section begins with setting up the context of musical rhythm within which the occurrence of beat and meter is encountered. Beat and meter are then discussed from both a musicological and behavioral point of view. The section concludes with providing a brief description of how beat and meter might be reflected in human's brain physiological function. A more detailed analysis in the section thereafter considers the physiological structures the model assumes and what, according to the model, happens when the brain confronts the musical situations of beat and meter described in the text below.

### Rhythm

Musical rhythm has been the subject of a large number of definitions among different scholars, musicians and enthusiasts, between different cultures and epochs. As Arom (1991) suggests, "There are so many meanings for this term that it would be useless to try to enumerate them all" (Arom, p.186). Despite this diversity, the aim of this section is to describe what musical rhythm means in terms of what the computational model tries to simulate. In that regard, musical rhythm accounts for the potential ways individual musical events can be temporally related with one another. More specifically, musical rhythm is used to refer to a temporal grid within which musical events can occur and relate with each other. The identification of musical rhythm as a temporal grid implies that individual musical events take place within a pattern of regularly spaced horizontal and vertical lines. Once musical events occur within this kind of rigidly structured organizational framework, a subsequent musical surface is created, which could then be subject of interpretation either tacitly or explicitly by human beings. One way human beings can interpret such musical surfaces is through the perceptual mechanisms of beat and meter. The following sections provide a description of these mechanisms in details.

**Beat**

The perception of beat in musical rhythm emerges as a perceptual organisation of related musical events in such a way that the overall progression gives rise to a perceived periodicity. In order to be perceived, this periodicity does not necessarily require all the adjacent musical events to unfold periodically or to be physically present. As Cooper and Meyer (1960) put it, "Though generally established and supported by objective stimuli (sounds), the sense of pulse (beat) may exist subjectively. A sense of pulse, once established, tends to be continued in the mind and musculature of the listener, even though the sound has stopped. For instance, objective pulses may cease or may fail for a time to coincide with the previously established pulse series. When this occurs, the human need for the security of an actual stimulus or for simplicity of response generally makes such passages point toward the re-establishment of objective pulses or to a return to pulse coincidence" (Cooper and Meyer, 1960, p.3).

The perceived periodicity of a musical surface can be used as a reference, which helps individuals to create expectations about when events are likely to occur in the future. Some common behavioural manifestations of such anticipation and predictability within the musical context are dancing to the music and tapping along with a song. Therefore, for one's synchronised motor responses to occur, a grasp of some rhythmic regularity (e.g. a sense of periodicity) of the music is required in the first place. Also, the fact that people respond differently in the exposure of the same musical surface, not only indicates that beat perception is subjective, but also that a single musical surface can be a subject of several beat attributions.

The following graph illustrates how a single musical surface can be subject of different beat attributions. We can think of each different level in the hierarchy as an indication of a foot taping along with a song or a person whose body movements can be reflected on more than one level at the same time.

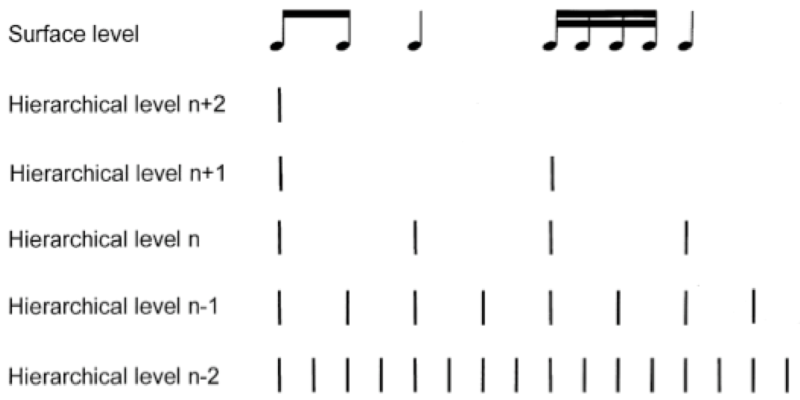



***Figure 1**: Hierarchical binary-branched representation of temporal structure. The vertical lines illustrate potential ways of beat perception in the presence of a rhythmical stimulus (i.e. surface level).1*

In the above graph, the potential ways of beat perception in regard to one specific musical surface is illustrated in a hierarchical way by using binary-branched hierarchical trees. The use of ternary branches is also allowed (Martin, 1972). The above proposition is based on the assertion that music is generally metrical, i.e. such kind of music has to have periodic structure at multiple time scales (Patel et al, 2005). Despite this, music can be non metrical too, which means that the organization of note events takes place in a less rigid and more elastic time framework, therefore attributing a beat in the sense it was described above it is not applicable. In this research context we are not considering non-metrical structures rather than we are focusing on metrically structured music.

In the above graph, potential ways of beat perception were pointed out in regard to one certain musical surface. A musical surface is much more complicated than the one single rhythmical line illustrated above. It is both vertically (harmonically) and horizontally (melodically) defined, with lots of different parameters such as timbre, pitch, duration and loudness. That means that each potential way of perceiving a beat could correspond to different explicitly present parameters or structural characteristics of a single musical surface. This

[1] Figure 1 derived from Drake et al. (2000,p.253)

parallel existence of beats and the interaction among them gives rise to the perception of meter.

**Meter**

Metrical structure can be described as the feeling of regularly recurring strong beats, which are also called *metrical accents* (Lerdahl & Jackendoff, 1983). This strength is not necessarily defined in terms of loudness, but rather it is more related to a phenomenal loudness. It is important to keep in mind that the phenomenal loudness of a certain beat could derive from different musical variables, such as pitch, loudness, timbre and duration.

This phenomenal loudness is based on the interaction among the potential beats of a single musical surface (Large, 2000). According to Large "metrical accents arise in temporal locations where the beats of many levels come into phase" (2000, p. 532). From that perspective, the perception of meter is not only based on the existence of several beats, but also on the fact that different beats might come into phase over the course of a musical piece. The hierarchical way of organizing musical events, which was described in the musical rhythm section above, can support such phase synchronization constraints, necessary for the establishment of meter. In other words, "a time point which is perceived as a beat on two different levels of pulsation is 'structurally stronger' than a point which is felt as a beat in only one level" (Clayton, p.31).

| Beats | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| Level 1 | . | . | . | . | . | . | . | . |
| Level 2 | . | | . | | . | | . | |
| Level 3 | . | | | | . | | | |

***Figure 2:*** *Lerdahl and Jackendoff's diagrammatic representation of metrical hierarchy based on several pulsations of beat. For example, musical events corresponding to level 2 would be felt stronger than the ones corresponding to level 1.*

London further suggests that any two levels of pulsation alone are not sufficient for the perception of meter, as Large's argument above might imply, but "at minimum, a metrical pattern requires a tactus (i.e. the most salient) coordinated with one other level of organisation" (2004, p.17).

**Perceptual correlates of Beat and Meter**

In the sections above, beat and meter are described as perceptual processes. In general, we could assert that perceptual mechanisms are reflected in brain activity. Therefore, we can expect beat and meter to be reflected in brain activity. In order to explain how beat and meter might be reflected in the brain we have to consider the following two propositions. Firstly, we have keep in mind the fact that brain activity exhibits periodic behaviour even in the absence of external rhythmic stimuli (Hoppensteadt & Izhikevich, 1996), and secondly, we have to assume that this periodic activity can be modulated by the rhythm of a musical surface. This section provides a series of theoretical and experimental insights according to which the assumed modulation of brain activity can be explained by the functions of the cognitive mechanism of attention. In this way, we establish an initial link to support the provisional proposition of how beat and meter as perceptual processes could be reflected in brain activity.

According to Jones (1981) the perception of beat is based on a dynamic process in which the temporal organisation of external musical events synchronises a listener's internal rhythmic processes. More specifically, Jones has proposed the *Dynamic Attending* theory according to which: "A rhythmical approach to attending is one which assumes that people and other animals target attending over predetermined time intervals toward events in space and time in a rhythmical fashion. Attending is an energistic activity guided in part by explicitly dynamic schemes that are themselves set in motion or indeed synchronously driven by the ongoing temporal character of an environment" (1986, p.19).

In a similar fashion London (2004, p. 4) defines meter as a form of synchronisation of some aspect of our biological activity with regularly recurring events in the environment. Meter is more, however, than just a stimulus driven form of synchronization. Metric behaviours are also learned – they can be rehearsed and practiced (London, 2004). Supportive to London's overall opinion about meter is Barnes and Jones' study in expectancy, attention, and time (2000).

According to that study, the implied biological synchronisation is rendered via the attentional resources of human, which in the case of music can be modulated by the music's intrinsic rhythmical structure. However, the factors that influence the allocation of the attentional resources are related to both the physical characteristics of the musical events and personal differences (e.g. level of expertise), something that coincides with London's suggestions in the beginning of the paragraph. In general, the effect of such factors can be categorised as bottom-up control of attention and top-down control of attention.

On the one hand, bottom-up control of attention is associated with the effects of the physical characteristics of the stimulus on attention. In this case, attention is attracted by the nature of the stimulus without any active decisions on the part of the listener. The manifestation of such an attraction in rhythm perception has been addressed in both behavioural studies and brain-imaging studies of rhythm perception, in terms of synchronization between rhythmical stimuli and behavioural responses (e.g. tapping), and synchronization between rhythmical stimuli and brain activity respectively (Handel, 1984; Barnes, 2000; Large, 2002; Pfordresher, 2003; Repp, 2005; Wilson, 2009; Ellis & Jones, in press).

On the other hand, "top-down control of attention is associated with the voluntary control of attention, which results from high-level cognitive processes that are associated with decisions that guide attention toward a goal over the long term" (Barnes et al., 2000: p.255). Typical manifestations of top-down control of attention in rhythm perception are the process of metrical interpretation or the shift of attention towards one of the rhythms in a polyrhythmic stimulus (Pressing et al. 1996). A brain study by Iversen (2009) has shown that attributing different metrical structures to the same musical surface can also be reflected in brain activity.

The above findings provide an explanation about the perceptual implications of beat and meter in terms of the cognitive mechanism of attention. Furthermore, the above findings provide support to the opening assumption of this section, where the rhythmic activity of the brain can be modulated and get synchronized to some rhythmical stimuli. Consequently, the following section seeks to ground the above proposition in the physiological reality of the human brain. In other words, the following section will provide a potential explanation of what happens to the brain when it confronts the musical situations of beat and meter such as those described in the sections above. These hypothesized physiological processes are then used to inform both the hypothesis of the computational model and the appropriate level of mathematical abstraction. These will be described in detail in the last section.

## 2.Hypothesised physiological processes

The first hypothesized physiological response when the brain confronts a periodic stimulus of a musical surface is that the firing rate of some of its neurons will synchronize to the frequency of the stimulus' beat. This assumption can be broken down into two legs. The first leg implies that inasmuch the beat of a musical stimulus is discrete and periodic, the brain activity has to be discrete and periodic too before the two can synchronise. In the opening paragraph of the previous section we mentioned that brain activity has been observed to be discrete and occasionally, periodic (Hodgin & Huxley, 1956; Shepherd, 1976). These empirical observations satisfy this first leg of the assumption, in the sense that if brain activity has to synchronise with a periodic and discrete stimulus, then the nature of this activity has to be discrete and periodic too.

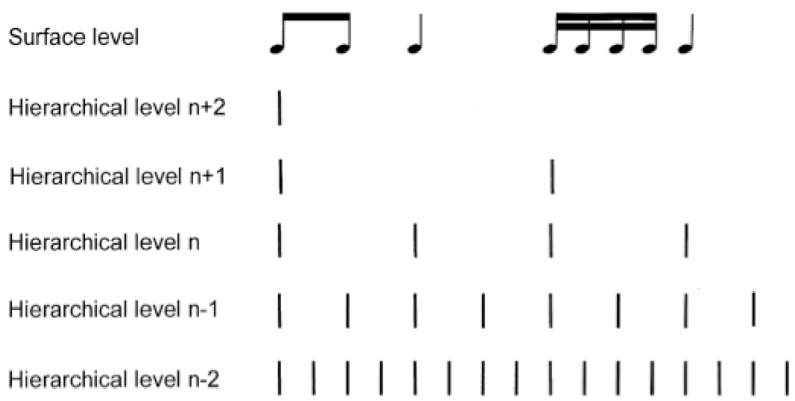
The second leg of the opening assumption suggests that the brain activity has also to synchronise with the stimulus's beat. We support this in two steps. The first step provides a mathematical explanation of how the behaviorally observed periodicity of brain activity even in the absence of a rhythmical stimulus might occur. The second step is more speculative but it is based on the same mathematical grounds of the explanation provided in the first step. This second step provides an explanation of how the synchronization between brain activity and stimulus can be succeeded.

According to Hoppensteadt & Izhikevich (1997) the mechanism through which brain's periodic firing occurs, is based on the interaction of local populations of opposing neurons. By opposing we mean that the two populations exhibit opposite functions; one population tries to increase the firing rate, while the other one tries to decrease the firing rate. This interaction will eventually lead the two populations to fire rhythmically. For reasons of simplicity, Hoppensteadt & Izhikevich considered just a pair of neurons, one from each population, and noted that they would behave like a pair of coupled oscillators. This pair of neurons has then been described as a *neural oscillator.*

Hoppensteadt and Izhikevich's correlation of individual neurons as oscillators can then be used to make the following assumption in order to address how the synchronization between brain activity and stimulus can occur. In the case of oscillators in general, when a periodic external signal drives an oscillator of a similar frequency, the latter will vibrate harmonically with the period of that force. Based on the above property of oscillators, and Hoppensteadt and Izhikevich's view of neurons as oscillators, we can assume a neural oscillator can get synchronized to an external rhythmical stimulus and therefore this mechanism might explain how humans perceive beat and meter.

In the above example it was assumed that the rhythmical stimulus is periodic. In musical surfaces, however, it is more likely that the adjacent events will not generally be periodic. Despite this non-periodicity, humans are still able to tap their foot along with such musical surfaces in a periodic fashion. In such cases, the hypothesized physiological processes to explain how the brain responds to a more realistic musical surface need to be a little more complex. In the following paragraph, we consider one candidate mechanism that might help to explain the response in this more complex case.

In the human brain, if we assume the existence of a bank of neural oscillators each with different natural frequencies, then different oscillators will be activated by different intrinsic periodicities of a musical surface similar to the one described in Figure 1 (illustrated again for convenience below). In this candidate explanation, for simplicity of exposition it is assumed that at least four events will need to be periodic before an intrinsic periodicity can activate an oscillator. So for example, by assuming that the musical surface shown below is continually repeated we can argue the following:



- The first periodicities that could activate an oscillator even in the first repetition of the musical surface (*see* surface level in the graph), could be those represented by the hierarchical level *n* and *n-2*, because of the four adjacent quarter notes and the four adjacent sixteenth note onsets respectively. We can assume that these periodicities will activate two different neural oscillators whose natural frequencies will equal to the periods of the stimuli, namely the periods corresponding to level *n* and *n-2*. Once activated, their firing rates will also be in synchrony with the stimuli's rate. If *a* is the frequency corresponding to level *n*, then *a/4* will be the frequency corresponding to level *n-2.*
- By the end of the second repetition of the surface level the periodicity represented by the hierarchical level *n+1* will activate another neural oscillator, whose natural frequency equals to *a/2.*
- By the end of the fourth repetition of the surface level the periodicity represented by the hierarchical level *n+2* will activate another neural oscillator, whose natural frequency equals *4a.*

The assumed ability of neural oscillators of different natural frequencies to respond and synchronise to the intrinsic periodicities of musical surfaces, suggests in outline a brain process that might account for the perception of beat and meter.

Finally, we examine how periodic behaviours occur even when a musical surface is yet more ambiguous in terms of underlying periodicities, and what the explanation might be in terms of brain activity. For example even within highly syncopated musical surfaces humans are still able to respond periodically either by tapping or even dancing. To illustrate this we work backwards by considering the musical surface illustrated above and focusing on the case of the *n-1* level, which was the only level unable to be supported by the aforementioned assumption, where the activation of an oscillator demands four periodic events.

The candidate mechanism for extracting metrical levels from a musical surface implies that each potential interpretation of the musical surface, which is illustrated by the different levels *n* in the graph above, must necessarily be grounded to a bank of neural oscillators firing in synchrony with the intrinsic periodicities (*see* hierarchical levels*)* of the musical surface. In other words, the candidate model pre-supposes that any interpretation of a musical surface should be mirrored by the firing activity of some neural oscillator. If we now assume that a human is exposed to the musical surface (*see* above graph) and decides to tap his foot on the periodicity indicated by the *n-1* level, that would automatically mean that a neural oscillator with a natural frequency of *2a* should be activated, even if on the surface level there were no such periodic events to support this. And yet, given the assumed constraint of four adjacent periodic events before an oscillator can be activated, the present candidate model outlined above cannot straightforwardly account for the ability to tap in such a way.

An explanation of how such a neural oscillator might be activated, even if there is no explicit external stimulus to activate it at the relevant metrical level, requires a more sophisticated candidate mechanism. The key to such a mechanism lies in both the nature of individual neural oscillators and the interactions among different neural oscillators. On very general principles, it is clear that neurons, neural oscillators (i.e. coupled neurons) and populations of neurons are interconnected and interact with one another so they can carry out a vast number of tasks.

This leads us to Edward Large's theory of Neural Resonance, which is the fundamental hypothesis for the development of his computational model of rhythm perception. According to his theory "when a network of neural oscillators, spanning a range of natural frequencies, is stimulated with a musical rhythm, a multifrequency pattern of oscillations is established, which could account for behaviourally observed aspects of rhythm perception" such as beat and meter (2008: p.15). This general idea is simpler, less arbitrary, more physiologically and computational plausible, and fits empirical results much better than the previous candidate explanation.

To understand more about Large's theory and model, and exactly how it differs from the simpler model we outlined above, it will be helpful to be aware of the difference between linear and non-linear mathematical models, which we will begin in the next section.

## 3. The mathematics of the model

Often when creating mathematical models to simulate situations encountered in the physical world an important issue that needs to be addressed in the first place is what would be the most appropriate level of mathematical abstraction for the purposes of a more representative simulation. In this research, the mathematical abstraction chosen to account for simulating aspects of rhythm perception based on Large's general notion is based on the mathematics of non-linear oscillators. Consequently a brief explanation about the differences between linearity and non-linearity is provided.

### Linear and Non-linear Systems

The nature of a linear system can be illustrated by a filter whose net response when stimulated by two signals is the arithmetic sum of the responses that would have been caused by each signal individually. This general property is known as the superposition property of linear systems. By contrast, non-linear systems do not satisfy the principle of superposition. As a special case of the difference, the output of a non-linear system is not directly proportional to its input. To illustrate this difference, consider a bank of passive band-pass filters, which is typically a linear system, and a bank of self-powered oscillators, which is generally a non-linear system. To situate our example even closer to our current concerns, consider the musical surface illustrated in Figure 1 being used repetitively as input for the two systems contrasted above. Both systems are capable of being used to respond to the intrinsic periodicities of such an input. As we saw in the previous section, in the case of the passive band-pass filter, the output of the system will produce responses that indicate periodicities encountered on levels n-2, n, n+1 and n+2, but not in level n-1, as long as this particular musical surface cannot satisfy the constraint of the at least four periodic events before a periodicity is extracted. In contrast, in the case of the bank of self-powered oscillators for reasons we will touch on in the next section, and which are explained in detail in (appropriate Large reference) the response will correspond to all the potential levels. Broadly speaking, this contrast is due to both the nature of each self-powered oscillation and the interconnection between oscillators –

although this is not the only reason for non-linearity in Large's model, as we will explore in the next section.

The difference between linear and non-linear systems described above can be concretely illustrated in the graph derived from an actual comparison of such contrasted systems. In this graph, two sinewaves of frequency f1 and f2 were fed into both a bank of passive band-pass filters (*see* dotted linear filter in figure 3) and a bank of self-powered oscillators (*see* canonical model in figure 3). As we can see the output of the bank of linear filter responds only in the frequencies that are present in the input signal, while the output of the non-linear filter produces responses in frequencies with no physical presence in the input signal. Note that the response produced by the normal form filter shown in the diagram can be ignored.

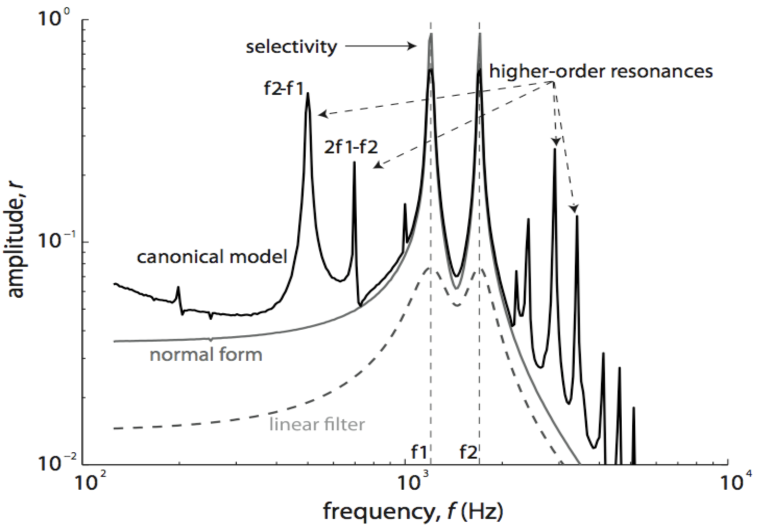



***Figure 3**: The output responses of both linear and non-linear (see canonical model) filters in the presence of an input signal consisting of two sinewaves of frequencies $f_1$ and $f_2$.*[2]

In the beginning of this section we said that the key level of mathematical abstraction to be adopted for simulating aspects of rhythm perception will be that of non-linear oscillators. The previous paragraph briefly outlined some aspects of non-linearity, and also outlined some differences between linear and non-linear systems.

2 Figure 3 derived from Large (in press)

We will now conclude this section by both recapping , and adducing new reasons, why this mathematical abstraction layer is so important for describing physiologically plausible models of human response to rhythm.

In a previous section we sketched how powered oscillators could provide simpler, less arbitrary, more physiologically plausible model of metrical perception than simpler approaches based on passive filters. Subsequently, in the present section, we asserted that the description of interlinked powered oscillators requires non-linear mathematics.

However, it turns out that non-linear mathematics are required not only for describing interconnected banks of neural oscillators, but also for describing the behaviour even of single neural oscillators. More particularly, a series of studies showed that certain aspects of neural behaviour at a range of levels could be explained in terms of non-linear systems. Firstly, Hodgin and Huxley (1952) used a set of non-linear differential equations to match experimental data obtained by the firing behaviour of one single giant squid's neuron. Secondly, Wilson and Cowan (1972) used a set of coupled non-linear differential equations to model the dynamics of spatially localised populations of model neurons. Thirdly, Hoppensteadt and Izhikevich (1996) used a weakly connected network of non-linear oscillators, to explain how observed rhythmic activity in the olfactory bulb, which is some area in the brain, occurs. The above findings are considered here as highly indicative of non-linearity in human brain. Just as a simple example to support the above proposition, the independent periodic activity of the neurons in the olfactory bulb area can be grounded to one of the fundamental properties of non-linear oscillators, which is their ability to independently exhibit periodic oscillations. In contrast, linear oscillators cannot exhibit independent periodic oscillation. Therefore, in modeling attempts of neural activity, non-linearity and non-linear oscillators in particular, seem to be repeatedly proven as the most appropriate level of mathematical abstraction for doing so.

## The computational model

As previously mentioned Large proposed the theory of neural resonance as providing a physiologically plausible mechanism to explain the various phenomena and suggested the use of mathematics of non-linear oscillators to model the theory. We can argue that this theory is physiologically plausible for the reasons described in the last paragraph of the previous section. The central assumption of the theory is that when a network of neural oscillators, spanning a range of natural frequencies, is stimulated with a musical rhythm, a multifrequency pattern of oscillations is established, which accounts for behaviourally observed aspects of rhythm perception (Large, 2008:15). The following section provides a detailed explanation of how to model a single neural oscillator based on the mathematics of a single non-linear oscillator.

## A single non-linear oscillator

The following equation (Fig. 4) has been previously used by Wilson & Cowan to model a single neural oscillator. In this equation three properties of a single non-linear oscillator will be discussed, which play essential roles in modeling the hypothesized physiology assumed by the neural resonance theory. These properties are:

- Spontaneous oscillation,
- Entrainment, and
- Higher order resonance.

$$\frac{dz}{dt} = z(\alpha + i\omega + (\beta + i\delta)|z|^2) + c\,s(t) + \text{h.o.t.}$$

***Figure 4:*** *A two-dimensional differential equation used to model a single neural oscillator*[3]

[3] Equation derived from Large, 2008- p.19

It would be convenient for purposes of exposition if we could start by drawing simple one to one correspondences between physical characteristics of neural oscillators and particular terms in the equation, but this is not straightforwardly possible. A more feasible approach is approach the equation in its own terms, and to draw correspondences between various aspects of its properties and the behaviour of neural oscillators as they become apparent.

The differential equation is two-dimensional, as $z$ is a complex variable, and is used to describe the state of a single non-linear oscillator. It has both real ($\alpha$, $\beta$) and imaginary ($\omega$, $\delta$) parameters, whose meaning will be explained below. The parameters are $\alpha$, the bifurcation parameter; $\beta$, the non-linear saturation parameter; ω, the eigenfrequency of the oscillator; and δ, the frequency detuning parameter. The bifurcation parameter ($\alpha$) determines the nature of the firing (i.e. either spontaneous oscillation or damped – see Fig. 5 below). The saturation parameter ($\beta$) prevents the amplitude from increasing indefinitely when $\alpha > 0$ (Fig. 5). The eigenfrequency (ω) is the natural frequency of the oscillator. The frequency detuning parameter (δ) describes how the instantaneous frequency of the oscillator also depends on its amplitude. The connection strength $c$ of a time-varying rhythmic stimulus $s(t)$ is a real number that describes the degree of dependency on an external stimulus. The stimulus s(t) consists of a series of discrete pulses, corresponding to the onset of individual events. It can be assumed that s(t)=1 at the onset of an event, and 0 at other times (Large & Kolen, 1994, p.13). The h.o.t. stands for *Higher Order Terms* which expands the equation to reveal further the non-linear nature of the oscillator. The non-linearity here mirrors how a single oscillator can stably entrain to various inputs whose periods form a simple ratio (e.g. 1:1, 2:1. 3:1, 3:2) with its natural frequency (period).

**Spontaneous oscillation**

The first property of a non-linear oscillator that we will focus on is the property of spontaneous oscillation. Spontaneous oscillation is the term that describes the stable oscillation that a single non-linear oscillator can exhibit. Spontaneous oscillation is one of the potential states of a non-linear oscillator.

This state is of particular interest for our purposes because it can be used to account for the exhibited stability of the perceived beat when responding to a stimulus, as described by Cooper & Meyer (1960, *p.4*). It also models the observed periodic firing of a neural oscillator in the absence or an external stimulus. Finally, it can model the persistence of the beat even when the musical surface has stopped.

Spontaneous Oscillation happens when the *bifurcation* parameter of the equation is positive ($\alpha > 0$). Figure 5 illustrates how the amplitude of the oscillator stabilises in the spontaneous oscillation state; in the absence of a stimulus (i.e. connection strength c is zero). Bifurcation can be thought of as the parameter responsible for controlling the transition between qualitatively different modes of oscillation (in this case spontaneous oscillation and damped oscillation). The critical point ($\alpha = 0$) is called Andronov-Hopf bifurcation. In cases where ($\alpha < 0$), the oscillator exhibits a damped oscillation, which means that the oscillation decays after some time (Fig. 5). In both cases, h.o.t. are ignored.

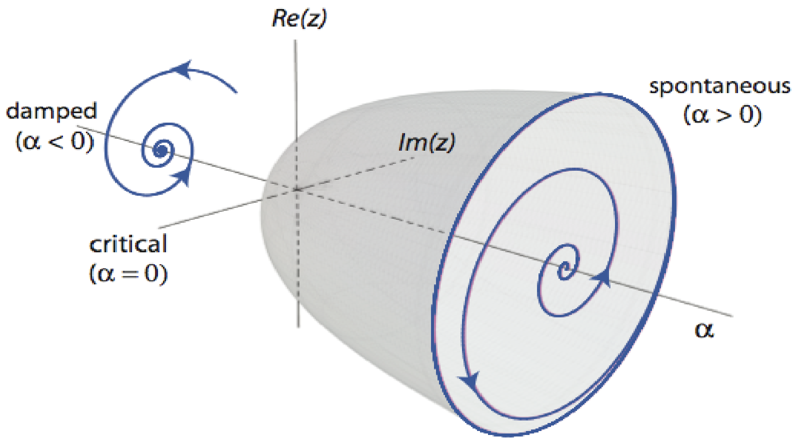



***Figure 5:*** *Trajectories of Spontaneous and Damped Oscillations. For a>0, the amplitude of the oscillator increases up to a saturation point, while for a<0 the amplitude decays to zero*[4]

[4] Figure derived from "Resonating to Rhythm" - Large, 2008 – p.21

The two states can be schematically understood by depicting the time-discrete states of the oscillations as a trajectory (Fig. 5). We can see that the spontaneous oscillation evolves to a stable phase, and repeats itself.

**Entrainment**

Entrainment is encountered in many disciplines such as biology, physics, and hydrodynamics and refers to the process whereby two interacting oscillating systems assume the same or related periods.

The property of entrainment of a single non-linear oscillator plays an essential role in modeling the assumed entrainment of a real neural oscillator to a rhythmical stimulus.

In the property of spontaneous oscillation, which is associated with the Wilson & Cowan equation, the oscillation can be independent of any stimulus and keep going even in the absence of a stimulus. However, in the case of entrainment the frequency of the oscillator ($\omega$) can be coupled to an external stimulus of some frequency close to $\omega$. In other words, the two oscillating systems interact and could phase lock with each other after some time. As Large states, "when a stimulus is present, spontaneous oscillation continues; however, stimulus coupling affects the oscillation's phase" (2008, p.21).

**Higher Order Resonance**

The full expansion of the equation that models a single non-linear oscillator is provided in the Figure 6 below. This equation expands to include the higher order terms mentioned in the previous equation. By incorporating these terms into the equation, a single non-linear oscillator can respond to rhythmic stimuli of various periods and not only to those that assume same or related periods.

$$\dot{z} = z(\alpha + i\omega + (\beta_1 + i\delta_1)|z|^2 + \frac{(\beta_2 + i\delta_2)\epsilon|z|^4}{1 - \epsilon|z|^2}) + c\mathcal{P}(\epsilon, x(t))\mathcal{A}(\epsilon, \bar{z})$$

***Figure 6:*** *Full expansion of the equation that describes a single non-linear oscillator* [5]

The parameters, $\omega$, $\alpha$, $\beta_1$ and $c$ correspond to the parameters of the not fully expanded model (Fig. 4). $\beta_2$ is an additional amplitude compression parameter, which acts as another limiter of the oscillator's amplitude. Two frequency detuning parameters $\delta_1$ and $\delta_2$ are new in this formulation, and make the oscillator's frequency dependent upon its amplitude. The last term of the equation introduces the behaviour of the oscillator, which is responsible of responding to several rhythmic inputs rather than only the one that shares a period similar to its own. Parameter $\varepsilon$ controls the amount of nonlinearity in the system. The coupling to a stimulus is nonlinear and has a passive part, P(ε, x(t)) and an active part, A(ε, z), which are responsible for producing nonlinear resonances, similar to those described in Figure 3 (p.16). This term accounts for the ability of the oscillator to entrain to inputs whose periods form a simple ratio, such as 1:1, 2:1, or 3:1, with its own period.

So far we have been describing one single non-linear oscillator, its properties of spontaneous oscillation, entrainment and higher order resonance, and the implications of these properties in modeling part of the assumed physiological realities proposed by the computational model. In fact, we have not yet described one of the most important characteristics of Large's model, which is the network of coupled non-linear oscillators. This will be the subject of the following section.

[5] Equation derived from Large (in press), p.9

**Network of non-linear oscillators**

Large's theory of neural resonance suggests that when a network of neural oscillators, spanning a range of natural frequencies, is stimulated with a musical rhythm, a multifrequency pattern of oscillations is established, which could account for behaviourally observed aspects of rhythm perception. It has been reported (Large, 2000) that for specific simulations, the periods of the oscillations were chosen to span the range from approximately 100 to 1500 ms, roughly corresponding to the range over which people accurately perceive changes in the tempo of auditory sequences (Drake & Botte, 1993). The period of each oscillator is represented by the natural frequency ($\omega$) of the oscillator.

The way in which those non-linear oscillators interact is by competing for activation through mutual inhibition. In other words, those oscillators that are the most consonant with the stimulus's frequency should tend to deactivate those (i.e. relatively dissonant oscillators) that do a poorer job of correlating with the incoming stimulus. So for example, we can assume that we have three non-linear oscillators with natural frequencies of 2Hz, 3Hz and 4Hz respectively. We then present (feed) a stimulus of 2Hz into the network of the oscillators. The first and the third oscillators have a more consonant relation with the stimulus's frequency compare to the one of the second oscillator. In this case, the first and the third oscillator will try to deactivate the second oscillator. This could be done by driving the amplitude $a$ of the dissonant oscillator below 0, therefore the oscillator will cease to oscillate (see Figure 5). Thus, in response to a rhythmic input signal, only a few oscillators should remain active, those that best reflect the temporal structure of the rhythm (Large, 2000, p.535).

To briefly summarise how beat and meter can be modeled using the mathematics of a network of coupled non-linear oscillators we can keep in mind that:

- The perception of musical beat can be modeled as an active, self sustained oscillation (spontaneous oscillation).
- The perception of meter can be modeled as a network of oscillators that are coupled to one another. Such a network should give rise to a dynamic

metrical percept whose structure reflects both the temporal structure of the input as well as internal dynamic constraints.

- Finally, the oscillators of the network, when driven with a complex external rhythm, should entrain to different periodicities of the stimulus's temporal spectrum. The entrainment of oscillators at multiple time scales resembles the listener's perceptual framework, based on which, the listener creates expectations about future events (Large & Kolen, 1994; Large and Jones, 1999).

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